تمرينات في أساسيات الحساب

1] أ. $Z = 3i^2 i - 6(i^2)^3 + 9i^8 i - 12(i^2)^6$

$Z = -3i + 6 + 9i - 12$

$Z = -6 + 6i$

الجواب "D"

2. $(Z + \bar{Z})^3 = (2x)^3$

$(Z + \bar{Z})^3 = (2.3)^3$

$(Z + \bar{Z})^3 = 216$

الجواب "C"

3. $(\bar{Z} - Z)^3 = [-(Z - \bar{Z})]^3$

$(\bar{Z} - Z)^3 = [-2yi]^3$

$(\bar{Z} - Z)^3 = [-2(2)i]^3$

$(\bar{Z} - Z)^3 = [-4i]^3$

$(\bar{Z} - Z)^3 = -64(-i)$

$(\bar{Z} - Z)^3 = 64i$

الجواب "A"

4. $|Z| = 2 \qquad |W| = 2$

$|Z + W|^2 - (Z\bar{W} + W\bar{Z})$

$(Z + W)(\bar{Z} + \bar{W}) - (Z\bar{W} - W\bar{Z})$

$= \underbrace{Z\bar{Z}} + \cancel{Z\bar{W}} + \cancel{W\bar{Z}} + \underbrace{W\bar{W}} - \cancel{Z\bar{W}} + \cancel{W\bar{Z}}$

$= |Z|^2 + |W|^2$

$= 4 + 4 = \boxed{8}$

الجواب "B"

5. $|W| = \left| \dfrac{-2iZ^2}{Z\bar{Z} + i} \right|$

Note: $Z\bar{Z} = |Z|^2 = 1$

$|W| = \left| \dfrac{-2iZ^2}{1 + i} \right|$

$|W| = \left| \dfrac{-2iZ^2}{1 + i} \right|$

$|W| = \dfrac{|-2i| \cdot |Z|^2}{\sqrt{1+1}} = \dfrac{\sqrt{0+4}\,(1)}{\sqrt{2}}$

$|W| = \dfrac{2}{\sqrt{2}} = \dfrac{\sqrt{2}\sqrt{2}}{\sqrt{2}} = \sqrt{2}$

الجواب "B"

6. لنختبر جميع الحالات...

$\bar{A} = \overline{\left( \dfrac{Z + \bar{Z}}{Z\bar{Z} + 2} \right)}$

$\bar{A} = \dfrac{\bar{Z} + Z}{\bar{Z}Z + 2} = A$

$\bar{A} = A$ محقق A

$\bar{B} = (\bar{Z} - Z)^2$

$\bar{B} = [-(Z - \bar{Z})]^2$

$\bar{B} = (Z - \bar{Z})^2 = B$

$\bar{B} = B$ محقق B

$\bar{C} = |\bar{Z}| + 2$

$\bar{C} = |Z| + 2 = C$

$\bar{C} = C$

حفظ...
$|Z| = |\bar{Z}|$

محقق C

$$\bar{D} = \frac{\bar{z}-z}{\bar{z}+z}$$

$$\bar{D} = \frac{-(z-\bar{z})}{z+\bar{z}} = -D$$

$$\bar{D} = -D$$

D تخيلية ...

الجواب "D".

[2] $l_1 = |z+w|^2 + |z-w|^2$

$= (z+w)(\bar{z}+\bar{w}) + (z-w)(\bar{z}-\bar{w})$

بالنشر...

$= 2z\bar{z} + 2w\bar{w}$

$= 2|z|^2 + 2|w|^2$

$= 2[|z|^2 + |w|^2] = l_2$

بتعرف لشو ......

لـ طلع عليه ٤٠ درجة

[3] $w = \frac{z - u\bar{z}}{1-u}$

$$\bar{w} = \frac{\bar{z} - \bar{u}z}{1-\bar{u}}$$

لكن $\bar{u} = \frac{|u|^2}{u}$

$\Rightarrow \bar{u} = \frac{1}{u}$

$$\bar{w} = \frac{\bar{z} - \frac{1}{u}z}{1-\frac{1}{u}}$$

$$= \frac{\left[\frac{u\bar{z}-z}{u}\right]}{\left[\frac{u-1}{u}\right]} = \frac{u\bar{z}-z}{u-1}$$

$$= \frac{-(z-u\bar{z})}{-(1-u)}$$

$$\bar{w} = \frac{z-u\bar{z}}{1-u} = w$$

w حقيقي ...

كمان عليه ٤٠ درجة ...

[4] $v = \frac{z+w}{1+zw}$

$$\bar{v} = \frac{\bar{z}+\bar{w}}{1+\bar{z}\bar{w}}$$

لكن $\bar{w} = \frac{1}{w}$ و $\bar{z} = \frac{1}{z}$

$$\bar{v} = \frac{\frac{1}{z}+\frac{1}{w}}{1+\frac{1}{zw}}$$

بالتوحيد والاختصار...

$$\bar{v} = \frac{w+z}{1+zw} = v$$

v حقيقي ...

[5] $w = z + \frac{4}{z}$ , $|z| = 2$

$$\bar{w} = \bar{z} + \frac{4}{\bar{z}}$$

لكن $|z| = 2$

$\Rightarrow \bar{z} = \frac{|z|^2}{z} = \frac{4}{z}$

$$\bar{w} = \frac{4}{z} + \frac{4}{\frac{4}{z}}$$

$$\bar{w} = \frac{4}{z} + 4\frac{z}{4} = \frac{4}{z} + z = w$$

$$\bar{w} = w$$

w حقيقي ...

6. $|w| = 3$ , $z = \frac{w^2+9}{iw}$

$\bar{z} = \frac{\bar{w}^2+9}{i\bar{w}}$

لكن $\bar{w} = \frac{|w|^2}{w}$

$\Rightarrow \bar{w} = \frac{9}{w}$

نعوض

$\bar{z} = \frac{(\frac{9}{w})^2+9}{-i\frac{9}{w}}$

$= \frac{\frac{81}{w^2}+9}{-i\frac{9}{w}} = \frac{\left[\frac{81+9w^2}{w^2}\right]}{-i\frac{9}{w}}$

$= \frac{9w^2+81}{-i9w} = \frac{9(w^2+9)}{-i9w}$

$= \frac{w^2+9}{-iw} = -z$

z تخيلي ...

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7. $|z+w| = |z-w|$

والمطلوب إثبات أن العدد $v = \frac{w}{z}$ تخيلي بحت...

نربع العلاقة الأساسية

$|z+w|^2 = |z-w|^2$

$(z+w)(\bar{z}+\bar{w}) = (z-w)(\bar{z}-\bar{w})$

$\cancel{z\bar{z}} + z\bar{w} + w\bar{z} + \cancel{w\bar{w}} = \cancel{z\bar{z}} - z\bar{w} - w\bar{z} + \cancel{w\bar{w}}$

$z\bar{w} + w\bar{z} = -z\bar{w} - w\bar{z}$

$2z\bar{w} = -2w\bar{z}$

* ... $\boxed{\bar{w} = \frac{-w\bar{z}}{z}}$

والآن...

$\bar{v} = \frac{\bar{w}}{\bar{z}}$

وبالاستعانة بالعلاقة * نجد...

$\bar{v} = \frac{\frac{-w\bar{z}}{z}}{\bar{z}} = \frac{-w\bar{z}}{z\bar{z}}$

$= -\frac{w}{z} = -v$

v تخيلي ...

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تمرينات زملائي والدرس

1. أ. $z = 3+\sqrt{3}i$

$x = 3 \qquad y = \sqrt{3}$

$r = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3}$

$\cos\theta = \frac{x}{r} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$

$\sin\theta = \frac{y}{r} = \frac{\sqrt{3}}{2\sqrt{3}} = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{6}$

$z = 2\sqrt{3}\, e^{i\frac{\pi}{6}}$

$= 2\sqrt{3}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

ب. $z = 1-\sqrt{3}i$

$x = 1 \qquad y = -\sqrt{3}$

$r = \sqrt{1+3} = \sqrt{4} = 2$

$\cos\theta = \frac{1}{2}$

$\sin\theta = \frac{-\sqrt{3}}{2}$

$\Rightarrow \theta = -\frac{\pi}{3}$

$z = 2e^{-i\frac{\pi}{3}}$

$= 2\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$

Memo

خلي بالك يا حباب انو

Memo

للتخلص من i المقام نضرب البسط والمقام بمرافق المقام

Memo

$$\frac{1}{a+ib} = \frac{1\,(a-ib)}{(a+ib)(a-ib)}$$

$$= \frac{a-ib}{a^2+b^2}$$

5

3 -

$z = \sqrt{2}\, i - \sqrt{6}$

$x = -\sqrt{6} \qquad y = \sqrt{2}$

$r = \sqrt{6+2} = \sqrt{8} = 2\sqrt{2}$

$\cos\theta = \frac{-\sqrt{6}}{2\sqrt{2}} = \frac{-\sqrt{3}}{2}$

$\sin\theta = \frac{\sqrt{2}}{2\sqrt{2}} = \frac{1}{2}$

$\Rightarrow \theta = \frac{5\pi}{6}$

$z = 2\sqrt{2}\, e^{i\frac{5\pi}{6}}$

$= 2\sqrt{2}\left(\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right)$

4 -

$z = \frac{1}{4} + \frac{\sqrt{3}}{4} i$

$x = \frac{1}{4} \qquad y = \frac{\sqrt{3}}{4}$

$r = \sqrt{\frac{1}{16} + \frac{3}{16}} = \sqrt{\frac{4}{16}} = \frac{1}{2}$

$\cos\theta = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$

$\sin\theta = \frac{\frac{\sqrt{3}}{4}}{\frac{1}{2}} = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{3}$

$z = \frac{1}{2}\, e^{i\frac{\pi}{3}}$

$= \frac{1}{2}\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)$

5-

$$z = \frac{1}{5+5i}$$

$$z = \frac{1\,(5-5i)}{(5+5i)(5-5i)} = \frac{5-5i}{50}$$

$$= \frac{5}{50} - \frac{5}{50}i = \frac{1}{10} - \frac{1}{10}i$$

$$x = \frac{1}{10} \qquad y = -\frac{1}{10}$$

$$r = \sqrt{\frac{1}{100} + \frac{1}{100}} = \sqrt{\frac{2}{100}} = \frac{\sqrt{2}}{10}$$

$$\cos\theta = \frac{\frac{1}{10}}{\frac{\sqrt{2}}{10}} = \frac{\sqrt{2}}{2}$$

$$\sin\theta = \frac{-\frac{1}{10}}{\frac{\sqrt{2}}{10}} = -\frac{\sqrt{2}}{2}$$

$$\theta = -\frac{\pi}{4}$$

$$z = \frac{\sqrt{2}}{10} e^{-i\frac{\pi}{4}}$$

$$z = \frac{\sqrt{2}}{10}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$$

6-

$$z = (\sqrt{2} + \sqrt{2}i)^2$$

$$z = 2 + 2\sqrt{2}i + (-2)$$

$$z = 4i$$

$$z = 4e^{i\frac{\pi}{2}}$$

7-

$$z = \sqrt{3} - 3$$

احـــذر السالب المخفي

$$z = -(3 - \sqrt{3})$$

$$z = (3 - \sqrt{3})e^{i\pi}$$

8-

$$z = -3$$

$$z = 3e^{i\pi}$$

9-

$$z = \frac{-i}{2}$$

$$z = -\frac{1}{2}(i)$$

$$z = \frac{1}{2} e^{-i\frac{\pi}{2}}$$

10-

$$z = (3+i)/(1+2i)$$

$$z = \frac{1+2i}{3+i}$$

نضرب بمرافق المقام...

$$z = \frac{(1+2i)(3-i)}{(3+i)(3-i)}$$

$$z = \frac{5+5i}{10} = \frac{5}{10} + \frac{5}{10}i$$

$$z = \frac{1}{2} + \frac{1}{2}i$$

$$x = \frac{1}{2} \qquad y = \frac{1}{2}$$

$$r = \sqrt{\frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$$

$$\cos\theta = \frac{\frac{1}{2}}{\frac{1}{\sqrt{2}}} = \frac{\sqrt{2}}{2}$$

$$\sin\theta = \frac{\frac{1}{2}}{\frac{1}{\sqrt{2}}} = \frac{\sqrt{2}}{2}$$

$$\boxed{\theta = \frac{\pi}{4} \qquad z = \frac{1}{\sqrt{2}} e^{i\frac{\pi}{4}}}$$

$\cos\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\sin\theta = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$

$\theta = -\frac{\pi}{4}$

$Z = \sqrt{2}\, e^{-i\frac{\pi}{4}}$

$Z = \sqrt{2}\left(\cos\left(\frac{-\pi}{4}\right) + i\sin\left(\frac{-\pi}{4}\right)\right)$

$Z = \frac{6i - 2}{1 - 3i}$ .i4

$Z = \frac{2(3i - 1)}{1 - 3i} = \frac{-2(\cancel{1 - 3i})}{\cancel{(1 - 3i)}}$

$Z = -2$

$x = -2$ $y = 0$

$r = \sqrt{4}$

$\cos\theta = \frac{-2}{\sqrt{4}} = -1$

$\sin\theta = \frac{0}{\sqrt{4}} = 0$

$\theta = \pi$

$Z = 2e^{i\pi}$

$Z = 2(\cos\pi + i\sin\pi)$

$Z = 3 + 3i^3$ .11.

$Z = 3 - 3i$

$x = 3$ $y = -3$

$r = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$

$\cos\theta = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\sin\theta = \frac{-3}{3\sqrt{2}} = \frac{-1}{\sqrt{2}}$

$\theta = \frac{-\pi}{4}$

$Z = 3\sqrt{2}\, e^{-i\frac{\pi}{4}}$

$Z = -2\sqrt{3} + 6i$ .i2

$x = -2\sqrt{3}$ $y = +6$

$r = \sqrt{12 + 36} = \sqrt{48} = 4\sqrt{3}$

$\cos\theta = \frac{-2\sqrt{3}}{4\sqrt{3}} = \frac{-1}{2}$

$\sin\theta = \frac{+6}{4\sqrt{3}} = \frac{\sqrt{3}}{2}$

$\theta = \frac{2\pi}{3}$

$Z = 4\sqrt{3}\, e^{i\frac{2\pi}{3}}$

$Z = 4\sqrt{3}\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)$

$Z = \frac{3 + i}{1 + 2i}$ .i3.

$Z = \frac{(3 + i)(1 - 2i)}{(1 + 2i)(1 - 2i)} = \frac{5 - 5i}{5}$

$Z = 1 - 1i$

$x = 1$ $y = -1$

$r = \sqrt{1 + 1} = \sqrt{2}$

2 أ

$w_1 = 1+\sqrt{3}i = 2e^{i\frac{\pi}{3}}$

$w_2 = 1-i = \sqrt{2}e^{-i\frac{\pi}{4}}$

$w_1^5 \Rightarrow r = (2)^5 = 32$

$\Downarrow \quad \theta = 5\left(\frac{\pi}{3}\right) = \frac{5\pi}{3}$

$$\boxed{w_1 = 32e^{i\frac{5\pi}{3}}}$$

$w_2^3 \Rightarrow r = (\sqrt{2})^3 = 2\sqrt{3}$

$\Downarrow \quad \theta = 3\left(-\frac{\pi}{4}\right) = \frac{-3\pi}{4}$

$$\boxed{w_2^3 = 2\sqrt{2}\, e^{i\frac{3\pi}{4}}}$$

$z = \frac{w_1^5}{w_2^3} \Rightarrow r = \frac{32}{2\sqrt{2}} = \frac{16}{\sqrt{2}}$

$\Downarrow \quad \theta = \frac{5\pi}{3} - \frac{-3\pi}{4} = \frac{29\pi}{12}$

$\Rightarrow z = \frac{16}{\sqrt{2}} e^{\frac{29\pi}{12}i}$

$\Rightarrow z = 8\sqrt{2}\, e^{i\frac{5\pi}{12}}$

3 الطلب الأول ...

$z_1 = \frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}i$

$r = \sqrt{\frac{6}{4} + \frac{2}{4}} = \sqrt{2}$

$\cos\theta = \frac{\frac{\sqrt{6}}{2}}{\sqrt{2}} = \frac{\sqrt{3}}{2}$

$\sin\theta = \frac{-\frac{\sqrt{2}}{2}}{\sqrt{2}} = -\frac{1}{2}$

$\theta = -\frac{\pi}{6}$

$z_1 = \sqrt{2}\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)$

$z_2 = 1 - i$

بنفس الطريقة السابقة نحوله لقطبي معنا بالشكل

$z_2 = \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$

$\frac{z_1}{z_2} \Rightarrow r = \frac{\sqrt{2}}{\sqrt{2}} = 1$

$\Downarrow \quad \theta = -\frac{\pi}{6} - \frac{-\pi}{4} = \frac{\pi}{12}$

$\Rightarrow \frac{z_1}{z_2} = 1\left[\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right]$

الطلب الثاني ...

$\frac{z_1}{z_2} = \frac{\frac{\sqrt{6}-\sqrt{2}i}{2}}{1-i}$

$= \frac{1}{2}\,\frac{\sqrt{6}-\sqrt{2}i\,(1+i)}{1-i\,(1+i)}$

$= \frac{1}{2}\,\frac{\sqrt{6}+\sqrt{6}i-\sqrt{2}i+\sqrt{2}}{2}$

ث.

$z_I = \frac{a+b}{2}$

$= \frac{2-\sqrt{3}+i}{2}$

$z_I = \frac{2-\sqrt{3}}{2} + \frac{1}{2}i$

بالمثلثي...

$r = \sqrt{\left(\frac{2-\sqrt{3}}{2}\right)^2 + \frac{1}{4}}$

$r = \sqrt{\frac{4-4\sqrt{3}+3}{4} + \frac{1}{4}}$

$r = \sqrt{\frac{8}{4} - \frac{4\sqrt{3}}{4}}$

$r_I = \sqrt{2-\sqrt{3}}$

$\theta_I = \frac{5\pi}{12}$

$z_I = \sqrt{2-\sqrt{3}}\; e^{i\frac{5\pi}{12}}$

مثلثي $z_I = z_I$ جبري

$\sqrt{2-\sqrt{3}}\left[\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right] = \frac{2-\sqrt{3}}{2} + \frac{1}{2}i$

$\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12} = \frac{2-\sqrt{3}}{2\sqrt{2-\sqrt{3}}} + i\frac{1}{2\sqrt{2-\sqrt{3}}}$

$= \frac{(\sqrt{6}+\sqrt{2}) + i(\sqrt{6}-\sqrt{2})}{4}$

$\frac{z_1}{z_2} = \frac{\sqrt{6}+\sqrt{2}}{4} + i\,\frac{\sqrt{6}-\sqrt{2}}{4}$

مثلثي $\left(\frac{z_1}{z_2}\right) = \left(\frac{z_1}{z_2}\right)$ جبري

$\cos\frac{\pi}{12} + i\sin\frac{\pi}{12} = \frac{\sqrt{6}+\sqrt{2}}{4} + i\,\frac{\sqrt{6}-\sqrt{2}}{4}$

$\cos\frac{\pi}{12} = \frac{\sqrt{6}+\sqrt{2}}{4}$

$\sin\frac{\pi}{12} = \frac{\sqrt{6}-\sqrt{2}}{4}$

$\boxed{4}$ أ.

$a = 2$ — $A$

$b = 2e^{i\frac{5\pi}{6}}$ — $B$

$I$ منتصف $[AB]$

$b = 2\left[\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right]$

$= 2\left[-\frac{\sqrt{3}}{2} + i\,\frac{1}{2}\right]$

$b = -\sqrt{3} + i$

$A(2,0)$ — $B \to r = 2$, $\to \frac{5\pi}{6}$ تذكير

ت.

$OA = OB = ?$

$OAB$ مثلث متساوي الساقين

$A\hat{O}B = ?$

$A\hat{O}I = \frac{1}{2} A\hat{O}B$

$= \frac{1}{2}\,\frac{5\pi}{6} = \frac{5\pi}{12}$

5) أ) • $z_1 = 1+\sqrt{3}i$

$z_1 = 2e^{i\frac{\pi}{3}}$ بالرسم

• $z_2 = e^{-i\frac{\pi}{4}}$

$= \cos\left(\frac{-\pi}{4}\right) + i\sin\left(\frac{-\pi}{4}\right)$

$z_2 = \frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$ بالجبري

ب) $l_1 = z_1 \cdot z_2 = 2e^{i\frac{\pi}{3}} \cdot e^{-i\frac{\pi}{4}}$

$= 2e^{i\left(\frac{\pi}{3}-\frac{\pi}{4}\right)} = 2e^{i\frac{\pi}{12}}$

$= 2z_3 = l_2$

جـ) $z_1 \cdot z_2 = 2z_3$

$z_3 = \frac{1}{2} z_1 \cdot z_2$

$= \frac{1}{2}(1+\sqrt{3}i)\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)$

$= \frac{1}{4}(1+\sqrt{3}i)(\sqrt{2} - \sqrt{2}i)$

$= \frac{1}{4}(\sqrt{2} - \sqrt{2}i + \sqrt{6}i + \sqrt{6})$

$= \frac{1}{4}\left((\sqrt{6}+\sqrt{2}) + i(\sqrt{6}-\sqrt{2})\right)$

$z_3 = \frac{\sqrt{6}+\sqrt{2}}{4} + \frac{\sqrt{6}-\sqrt{2}}{4}i$

د) جبري $z_3 = z_3$ مثلثي

$\cos\frac{\pi}{12} + i\sin\frac{\pi}{12} = \frac{\sqrt{6}+\sqrt{2}}{4} + \frac{\sqrt{6}-\sqrt{2}}{4}i$

$\cos\frac{\pi}{12} = \frac{\sqrt{6}+\sqrt{2}}{4}$

$\sin\frac{\pi}{12} = \frac{\sqrt{6}-\sqrt{2}}{4}$

علاقات أويلر ... س٦

1) أ) $z = \frac{\cos x + i\sin x}{\cos x - i\sin x}$

$= \frac{e^{ix}}{e^{-ix}} = e^{i2x}$

الجواب "B"

ب) $z = \frac{1+\cos\theta - i\sin\theta}{1+\cos\theta + i\sin\theta}$

$= \frac{1+e^{-ix}}{1+e^{ix}}$

$= \frac{1+\frac{1}{e^{ix}}}{1+e^{ix}}$

نوحد المقامات في البسط

$z = \frac{\left[\frac{\cancel{e^{ix}+1}}{e^{ix}}\right]}{\cancel{1+e^{ix}}} = \frac{1}{e^{ix}}$

$z = e^{-ix}$

الجواب "B"

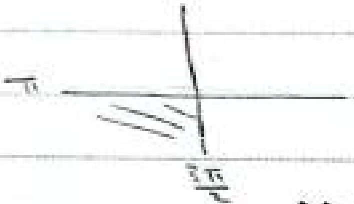
جـ) $z = \frac{e^{i\theta} - e^{-i\theta}}{e^{i\theta} + e^{-i\theta}}$

$z = \frac{2i\sin\theta}{2\cos\theta} = i\tan\theta$ الجواب "A"

[1]

$$Z = 1 + e^{i2\theta}$$
$$Z = e^{i\theta}\left[\frac{1}{e^{i\theta}} + e^{i\theta}\right]$$
$$Z = e^{i\theta}\left(e^{i\theta} + e^{-i\theta}\right)$$
$$Z = \underbrace{2\cos\theta}\ .\ e^{i\theta}$$

حقيقي موجب؟؟!!!!




للأسف؟؟

$2\cos\theta$ سالب ... بدو علاج...

$$-(-2\cos\theta)$$
$$-e^{i\theta}\left(\underbrace{-}\ \underbrace{2\cos\theta}\right)$$

سالب . سالب = موجب :)

نعوض..

$$Z = (-2\cos\theta)\, e^{i\pi}\, e^{i\theta}$$
$$Z = (-2\cos\theta)\, e^{i(\pi+\theta)}$$
$$r = -2\cos\theta$$
$$\arg(Z) = \pi + \theta$$

[2]

$$Z = 1 + e^{i\frac{2\pi}{3}}$$
$$Z = e^{i\frac{\pi}{3}}\left[\frac{1}{e^{i\frac{\pi}{3}}} + e^{i\frac{\pi}{3}}\right]$$
$$= e^{i\frac{\pi}{3}}\left[e^{-i\frac{\pi}{3}} + e^{i\frac{\pi}{3}}\right]$$
$$= e^{i\frac{\pi}{3}}\left[e^{i\frac{\pi}{3}} + e^{-i\frac{\pi}{3}}\right]$$
$$= e^{i\frac{\pi}{3}}\ .\ \underbrace{2\cos\left(\frac{\pi}{3}\right)}_{=1}$$
$$Z = 1\, .\, e^{i\frac{\pi}{3}}$$

ملاحظة:

$$e^{i\theta} + e^{-i\theta} = 2\cos\theta$$

[3]

$$Z = e^{i\frac{\pi}{4}} - 1$$
$$Z = e^{i\frac{\pi}{8}}\left[e^{i\frac{\pi}{8}} - \frac{1}{e^{i\frac{\pi}{8}}}\right]$$
$$Z = e^{i\frac{\pi}{8}}\underbrace{\left[e^{i\frac{\pi}{8}} - e^{-i\frac{\pi}{8}}\right]}_{=2i\sin\frac{\pi}{8}}$$
$$Z = \underbrace{2i\sin\frac{\pi}{8}}\ .\ e^{i\frac{\pi}{8}}$$

حقيقي موجب؟؟!!!!

$$= 2e^{i\frac{\pi}{2}}\sin\frac{\pi}{8}\ .\ e^{i\frac{\pi}{8}}$$
$$= 2\underbrace{\left(\sin\frac{\pi}{8}\right)}\, e^{i\left(\frac{5\pi}{8}\right)}$$

موجبة؟!!

نعم لأن $\frac{\pi}{8}$ في الربع الأول

[5] معلوم ...

$z_1 . z_2 = e^{ix} e^{iy} = e^{i(x+y)}$

$z_1 . z_2 = \cos(x+y) + i\sin(x+y)$ ... [1]

$z_1 . z_2 = e^{ix} e^{iy}$

$= (\cos x + i\sin x)(\cos y + i\sin y)$

$= \cos x \cos y + i\sin y \cos x + i\sin x . \cos y - \sin x \sin y$

$z_1 . z_2 = \cos x \cos y - \sin x \sin y + i[\sin x . \cos y + \sin y \cos x]$ ... [2]

بمقارنة العلاقة [1] والعلاقة [2] ...

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\sin(x+y) = \sin x \cos y + \sin y \cos x$

Nehad S A B B A G H

... الجذور ...

وهي نوعان ...

1) الجذور التربيعية $\boxed{\sqrt{a+ib}}$ ...

$x^2 + y^2 = r = \sqrt{a^2+b^2}$

$x^2 - y^2 = a$

$2xy = b$

أجمع 1 و 2 ...

$2x^2 = \heartsuit$

$x_1$ ← ... → $x_2$

نعوض في المعادلة الثالثة

$y_1$ ... $y_2$

$w_1 = x_1 + iy_1$ ... $w_2 = x_2 + iy_2$

2) الجذور التكعيبية ...

$Z^3 = a + ib$

$(r e^{i\theta})^3 = \alpha e^{i\beta}$

$r^3 e^{i3\theta} = \alpha e^{i\beta}$

$r^3 = \alpha$

$3\theta = \beta + 2\pi k$

$Z_0 \quad / \quad Z_1 \quad / \quad Z_2$

السؤال الأول ...

حول الجذور لسكين $w=8$ , وأوجد الجذور التكعيبية للعدد 8 ...

الحل $\quad Z^3 = 8$ نجمع

$(r e^{i\theta})^3 = 8 e^{i(0)}$

$r^3 e^{i3\theta} = 8 e^{i(0)}$

$\Rightarrow r^3 = 8$ و $3\theta = 0 + 2\pi K$

$\Rightarrow r = 2$ و $\theta = \frac{2\pi k}{3}$

• $z_0 = 2e^{i(0)} \Rightarrow a = 2$

• $z_1 = 2e^{i(\frac{2\pi}{3})} \Rightarrow b = 2[\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}]$

$= 2[-\frac{1}{2} + i\frac{\sqrt{3}}{2}]$

$= -1 + i\sqrt{3}$

• $z_2 = 2e^{i(\frac{4\pi}{3})} \Rightarrow c = -1 - i\sqrt{3}$

2) • $a + b + c = ?$

$2 - 1 + i\sqrt{3} - 1 - i\sqrt{3} = 0$

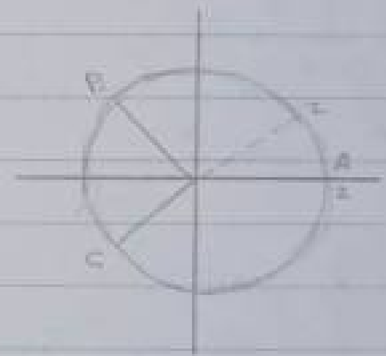
ملاحظة...

إذا كان مجموع الجذور التكعيبية معدوم ... هذا يعطي نتيجة هامة ألا وهي...

$\frac{a+b+c}{3} = 0$

⇐ O مركز ثقل ABC

3)




$|z_0| = |z_1| = |z_2| = 2$

A , B , C تقع على دائرة مركزها O

ونصف قطرها $r=2$.

4) O مركز ثقل ABC وهو مركز الدائرة المارة برؤوسه

⇐ متساوي الأضلاع.

السؤال الثاني...

1) $w = 3 + 4i$

$x^2 + y^2 = r$

$x^2 - y^2 = a$

$2xy = b$

$x^2 + y^2 = \sqrt{9+16} = 5$

$x^2 - y^2 = 3$

$2xy = 4$

نجمع 1 و 2...

$2x^2 = 8 \Rightarrow x^2 = 4$

$x = 2$ نعوض: $\Rightarrow 4y = 4 \Rightarrow y = 1$

$x = -2 \Rightarrow y = -1$

وهذا يعطي...

$w_1 = 2 + 1i$ و $w_2 = -2 - 1i$

2) $w = -20 - 21i$

$x^2 + y^2 = \sqrt{400 + 441} = \sqrt{841} = 29$

$x^2 - y^2 = -20$

$2xy = -21$

نجمع 1 و 2...

$2x^2 = 9 \Rightarrow x^2 = \frac{9}{2}$

$x = \frac{3}{\sqrt{2}}$ ; $x = \frac{-3}{\sqrt{2}}$

نعوض كل قيمة في المعادلة 3

$y = \frac{-7}{\sqrt{2}}$ ; $y = \frac{7}{\sqrt{2}}$

$w_1 = \frac{3}{\sqrt{2}} - \frac{7}{\sqrt{2}}i$ و $w_2 = \frac{-3}{\sqrt{2}} + \frac{7}{\sqrt{2}}i$

السؤال الثالث...

. المعادلات من الشكل $z^2 = a + ib$

. نحل على rab ...

• $z^2 = 6 + 8i$

إيجاد الجذور التربيعية

$w = 6 + 8i \Rightarrow z^2 = 6 + 8i$

$x^2 + y^2 = \sqrt{36+64} = 10$

$x^2 - y^2 = 6$

$2xy = 8$

... أتمم الحل وفقاً للمراحل السابقة...

• $z^2 - 2(1+\sqrt{2})z + 2(\sqrt{2}+2) = 0$

$a = 1 \qquad b = -2(1+\sqrt{2})$

$c = 2(\sqrt{2}+2)$

$\Delta = b^2 - 4a.c$

$= 4(1+\sqrt{2})^2 - 4(1)(2(\sqrt{2}+2))$

$= 4(1+2\sqrt{2}+2) - 8\sqrt{2} - 16$

$= 4 + 8\sqrt{2} + 8 - 8\sqrt{2} - 16$

$\Rightarrow \Delta = -4 < 0$

$\Rightarrow \Delta = 4i^2$

يوجد حلان عقديان مترافقان حيث

$a, b, c$ لا تحوي عدد حقيقي

$z_1 = \dfrac{-b+\sqrt{\Delta}}{2a} = \dfrac{2(1+\sqrt{2})+2i}{2}$

$\Rightarrow z_1 = \dfrac{2+2\sqrt{2}+2i}{2}$

$\Rightarrow z_1 = (1+\sqrt{2})+i$

بينما الحل الثاني بنفس الخطوات لكن بدل

$\sqrt{\Delta}$ نضع $-\sqrt{\Delta}$ فيصبح الحل...

$\Rightarrow z_2 = (1+\sqrt{2})-i$

• $z^2 - (2\cos\alpha)z + 1 = 0$

$a = 1 \qquad b = -2\cos\alpha \qquad c = 1$

$\Delta = b^2 - 4a.c$

$\Delta = 4\cos^2\alpha - 4 = 4[\cos^2\alpha - 1]$

$\Rightarrow \Delta = -4[1 - \cos^2\alpha]$

$\Delta = -4\sin^2\alpha < 0$

$\Delta = i^2 4\sin^2\alpha$

للمعادلة حلان عقديان مترافقان...

$z_1 = \dfrac{-b+\sqrt{\Delta}}{2a} = \dfrac{2\cos\alpha + 2i|\sin\alpha|}{2}$

$z_1 = \cos\alpha + i|\sin\alpha|$

$z_2 = \cos\alpha - i|\sin\alpha|$

* سؤال دورة 2021 ...

$z^2 + 2(1+i)z + i + \frac{3}{4} = 0$

$a = 1 \qquad b = 2(1+i) \qquad c = i + \frac{3}{4}$

$\Delta = 4(1+i)^2 - 4(i + \frac{3}{4})$

$= 4(1 + 2i - 1) - 4i - 3$

$= 8i - 4i - 3$

$\Delta = -3 + 4i$

ملاحظة...

عندما يكون $\Delta = a + ib$ نجري الحل كالآتي...

• المرحلة 1. إيجاد جذور $\Delta$ على rab فنحصل على $w_1, w_2$

• المرحلة 2. حل المعادلة

$z_1 = \dfrac{-b+w_1}{2a} \qquad z_2 = \dfrac{-b+w_2}{2a}$

لنطبق المراحل...

المرحلة أ- إيجاد جذر Δ...

$x^2+y^2=\sqrt{9+16}=5$

$x^2-y^2=-3$

$2xy=4$

نجمع ١ و 2...

$2x^2=2 \Rightarrow x^2=1$

$x=1$ | $x=-1$

نعوض عن كلا الحلين في المعادلة الثالثة

$y=2$ | $y=-2$

$w_1=1+2i$ | $w_2=-1-2i$

المرحلة ٢-

$\cdot\ z_1=\frac{-b+w_1}{2a}=\frac{-2(1+i)+1+2i}{2}$

$\Rightarrow z_1=-\frac{1}{2}$

$\cdot\ z_2=\frac{-b+w_2}{2a}=\frac{-2(1+i)-1-2i}{2}$

$\Rightarrow z_2=\frac{-3-4i}{2}$

---

* خواص $z_1$ و $z_2$...

$z_1+z_2=\frac{-b}{a}$

$z_1 \cdot z_2=\frac{c}{a}$

يعني الحلين مترافقين | يعني حل

الأمثال a,b,c | ومرافقه الآخر

السؤال السادس... ص ١٩

$z_1=1-2i$ $\quad z^2+pz+q=0$

$z_2=1+i$

$\cdot\ z_1+z_2=\frac{-b}{a} \Rightarrow 1-2i+1+i=\frac{-p}{1}$

$2-i=-p$

$p=-2+i$

$\cdot\ z_1\cdot z_2=\frac{c}{a} \Rightarrow (1-2i)(1+i)=\frac{q}{1}$

$q=1+i-2i+2$

$q=3-i$

السؤال الأول... ص ١٩

$a=2-2i$

$b=-1+7i$

$c=4+2i$

$d=-4-2i$

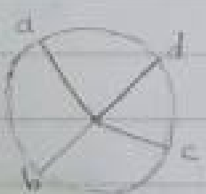
1)- $w=-1+2i$

$\Omega A=\sqrt{(3)^2+(-4)^2}=5$

$\Omega B=\sqrt{(0)^2+(5)^2}=5$

$\Omega C= \quad =5$

$\Omega D= \quad =5$

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السؤال الثاني...

- $a = 2$

$b = 1 + i\sqrt{3}$

$c = -1 + i\sqrt{3}$

$$\frac{a-b}{c-b} = \frac{2-1-i\sqrt{3}}{-1+i\sqrt{3}-1-i\sqrt{3}}$$

$$= \frac{1-i\sqrt{3}}{-2}$$

$$= -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$r = \sqrt{\frac{1}{4}+\frac{3}{4}} = 1$ $\quad \cos\theta = -\frac{1}{2}$ $\quad \sin\theta = \frac{\sqrt{3}}{2}$

$$\frac{a-b}{c-b} = e^{2i\frac{\pi}{3}}$$

$\left|\frac{a-b}{c-b}\right| = 1$ $\qquad \arg\left(\frac{a-b}{c-b}\right) = \frac{2\pi}{3}$

متساوي الساقين $\qquad$ منفرج الزاوية

السؤال السادس...

$a = 2+3i$ $\qquad b = 1-i$

$c = 3i$

$a' = 1+i$ $\qquad b' = 5+5i$

$c' = -3-i$

1) $\vec{AA'} + \vec{BB'} + \vec{CC'}$

$a'-a + b'-b + c'-c$

$a'+b'+c' - (a+b+c)$

$3+5i - (3+5i) = 0$

$$z_{G(A'B'C')} = \frac{a'+b'+c'}{3} = \frac{3+5i}{3}$$

2

$$e = \frac{a+b}{2} = \frac{1+5i}{2}$$

$$\frac{a-e}{d-e} = \frac{c-e}{a-e}$$

$$\ell_1 = \frac{2-2i-\frac{1+5i}{2}}{-4-2i-\frac{1+5i}{2}}$$

$$= \frac{\frac{4-4i-1-5i}{2}}{\frac{-8-4i-1-5i}{2}}$$

$$= \frac{3-9i}{-9-9i} \begin{matrix}\div 3\\ \div 3\end{matrix}$$

$$= \frac{1-3i}{-3-3i} \cdot \frac{(-3+3i)}{(-3+3i)}$$

$$= \frac{-3+3i+9i+9}{9+9} = \frac{6}{18} + \frac{12}{18}i$$

$$\ell_1 = \frac{1}{3} + \frac{2}{3}i$$

$$\ell_2 = \frac{c-e}{a-e} = \frac{4+2i-\frac{1+2i}{2}}{2-2i-\frac{1+5i}{2}}$$

$$= \frac{\frac{8+4i-1-2i}{2}}{\frac{4-4i-1-5i}{2}} = \frac{7+2i}{3-9i}$$

نعيد اللي عملناه فوق...

$$\ell_2 = \frac{1}{3} + \frac{2}{3}i$$

وبعد ذلك...

$$\ell_1 = \ell_2$$

$$\frac{a-e}{d-e} = \frac{c-e}{a-e}$$

$\hat{DEC}$ منصف للزاوية EA.


Nehad

3)

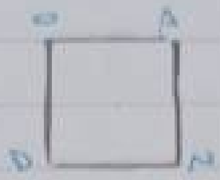
$OAND$

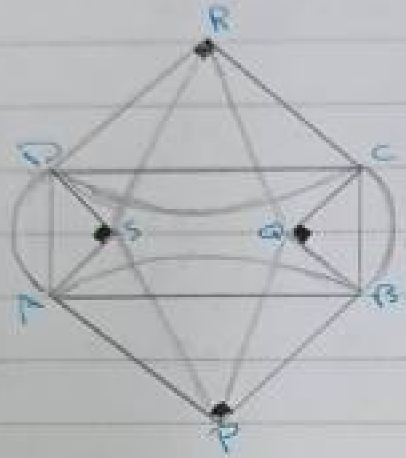
$\vec{OA} = \vec{DN}$

$a - o = n - d$

$n = a + d$

$n = 7 + 5i$




أ) R الدوران الذي مركزه ω ويقرن $z'$ بالعدد $z$ زاويته $\frac{\pi}{2}$ احسب ω بدلالة $z'$ و $z$

$z' - \omega = e^{i\frac{\pi}{2}}(z - \omega)$

$z' - \omega = iz - i\omega$

$z' - iz = \omega - i\omega$

$z' - iz = (1 - i)\omega$

$\omega = \frac{z' - iz}{1 - i} \cdot \frac{(1+i)}{(1+i)} = \frac{z' + iz' - iz + z}{2}$

$\Rightarrow \omega = \frac{(1-i)z + (1+i)z'}{2}$

قانون حساب مركز الدوران زاويته $\frac{\pi}{2}$

بالنسبة للمثلث ABP...

$p = \frac{(1-i)b + (1+i)a}{2}$

بالنسبة للمثلث BQC...

$q = \frac{(1-i)b + (1+i)c}{2}$

$z_{G(ABC)} = \frac{a+b+c}{3} = \frac{3+5i}{3}$

$\frac{b-a}{c-a} = k = 3$


$b - a = 3(c - a)$

$\vec{AB} = 3\vec{AC}$

$\underset{\text{صورة}}{z'} - \underset{\text{مركز}}{\omega} = \left[ k\, e^{i\theta} \right] (\underset{\text{أصل}}{z} - \underset{\text{مركز}}{\omega})$

$z' - \omega = e^{i\theta}(z - \omega)$

السؤال الرابع... صدوا...

$a = 6 - i$

$b = -6 + 3i$

$c = -18 + 7i$

أ) $\frac{b-a}{c-a} = \frac{-12 + 4i}{-24 + 8i}$

$= \frac{4(-3+i)}{8(-3+i)} = \frac{4}{8} = \frac{1}{2} \in \mathbb{R}$

$\Rightarrow$ A, B, C على استقامة واحدة.

ب) $d = 1 + 6i$ صورة $a$ وفق دوران مركزه (O) زاويته $\theta$ احسب $\theta$.

$z' - \omega = e^{i\theta}(z - \omega)$

$d - o = e^{i\theta}(a - o)$

$d = e^{i\theta} \cdot a$

$e^{i\theta} = \frac{d}{a} = \frac{1+6i}{6-i} \cdot \frac{(6+i)}{(6+i)} = \frac{37i}{37}$

$e^{i\theta} = i \Rightarrow e^{i\theta} = e^{i\frac{\pi}{2}} \Rightarrow \theta = \frac{\pi}{2}$

$\arg\left(\frac{i}{z}\right)^2 = -\frac{\pi}{3}$ (4)

$2\arg\left(\frac{i}{z}\right) = -\frac{\pi}{3}$

$\arg i - \arg z = -\frac{\pi}{6}$

$\frac{\pi}{2} - \arg z = -\frac{\pi}{6}$

$\arg z = \frac{\pi}{2} + \frac{\pi}{6}$

$\arg z = \frac{2\pi}{3}$

$Re(z + i\bar{z}) = 1$

$Re(x + iy + i(x - iy)) = 1$

$Re(x + iy + ix + y) = 1$

$Re((x + y) + i(x + y)) = 1$

$x + y = 1 \Rightarrow y = -x + 1$

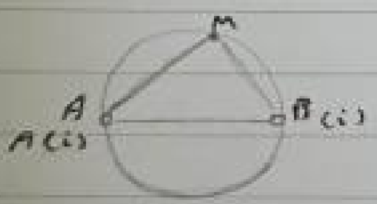
$xy = A$

$A = 0$ $\qquad$ $A \neq 0$

$xy = 0$ $\qquad$ $y = \frac{A}{x}$

$x = 0$ $\qquad$ $y = 0$

$\arg\left(\frac{z - i}{z - 1}\right) = \frac{\pi}{2}$




بالنسبة للمثلث ASO...

$s = \frac{(1-i)d + (1+i)a}{2}$

بالنسبة للمثلث DRC...

$r = \frac{(1-i)d + (1+i)c}{2}$

$p + r = s + q$ ??

(5) $p + r = \frac{(1+i)a + (1-i)b + (1+i)c + (1-i)d}{2}$

$s + q = \frac{(1+i)a + (1-i)b + (1+i)c + (1-i)d}{2}$

$p + r = s + q$

$p - s = q - r$

$\overrightarrow{SP} = \overrightarrow{RQ}$

$|z - a| = |z - b|$ (1)

محور القطعة $[AB]$

$|z - a| = \text{Const}$ (2)

لأحوى $z$

دائرة مركزها A نصف قطرها Const

$\arg(z) = 0$ (3)

نصف المستقيم الذي يصنع زاوية 0 مع محور الفواصل نصف مستقيماً...

$|z - 3i| = |3 + 4i|$ (2)

$|z - 3i| = \sqrt{9 + 16} = 5$

$|z| = 1$ $\qquad$ $|z - 0| = 1$ (3)

دائرة مركزها المبدأ، نصف قطرها 1

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$$Z = 3\left[\sin\frac{\pi}{3} + i\cos\frac{\pi}{3}\right]^5$$

$$= 3\left[\cos\left(\frac{\pi}{2}-\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{2}-\frac{\pi}{3}\right)\right]^5$$

$$= 3\left[\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right]^5$$

$$= 3\left[\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right]$$

$P(z) = z^3 - z^2 + z - 1$

احسب $P(1)$

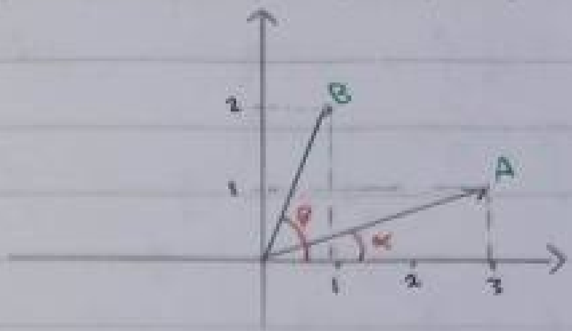
$P(1) = 1 - 1 + 1 - 1 = 0$

على $P(z)$ $z^2+1$

$z - 1$

$0 \quad 0$

$\Rightarrow (z-1)(z^2+1)$




احسب $\beta - \alpha$

$\alpha = \arg(Z_{\vec{OA}})$

$\beta = \arg(Z_{\vec{OB}})$

$$\beta - \alpha = \arg(Z_{\vec{OB}}) - \arg(Z_{\vec{OA}})$$

$$= \arg\left(\frac{b-0}{a-0}\right)$$

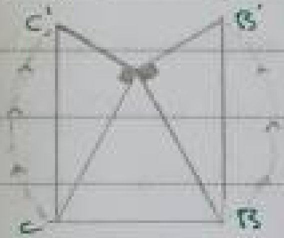
$$= \arg\left(\frac{1+2i}{3+i}\right)$$

$$= \arg\left(\frac{1}{2} + \frac{1}{2}i\right)$$

$$\beta - \alpha = \frac{\pi}{4}$$

مسألة ..

لتكن $(A, \vec{u}, \vec{v})$




$\bullet\ b' - a = e^{i\frac{\pi}{2}}(b - a)$

$b' = ib$

$\bullet\ c' - a = e^{-i\frac{\pi}{2}}(c - a)$

$c' = -ic$

$$\frac{c' - b}{c - b'} = \frac{-ic - b}{c - ib} \times i = \frac{(-ic - b)i}{(ic + b)}$$

$$= \frac{-(ic+b)i}{(ic+b)} = -i = e^{-i\frac{\pi}{2}}$$

$(C'B) \perp (CB')$

$C'B = CB'$

Nehad S A B B A G H

A مرجح $(C,1)$

$(B,1)$ $(C',3)$ $(B',2)$

$1\vec{AC} + 1\vec{AB} + 3\vec{AC'} + 2\vec{AB'} = \vec{0}$

$c-a+b-a+3(c'-a)+2(b'-a)=0$

$c+b+3c'+2b'=0$

$c+b-3ic+2ib=0$

$\frac{c}{b}+1-3i\frac{c}{b}+2i=0$ تقسيم b

$(1-3i)\frac{c}{b}=-1-2i$

$\frac{c}{b}=\frac{-1-2i}{1-3i}\cdot\frac{(1+3i)}{(1+3i)}$

$\frac{c}{b}=\frac{-1-3i-2i+6}{1+9}$

$\frac{c}{b}=\frac{1}{2}-\frac{1}{2}i$

$(o,\vec{u},\vec{v})$


ii. جد $e, b, d$

B صورة A وفق دوران مركزه $(o)$

بزاوية $\frac{\pi}{2}$

$b-o=e^{i\frac{\pi}{2}}(a-o)$

$b=ia$

D صورة C وفق دوران مركزه $(o)$

بزاوية $\frac{\pi}{2}$

$d-o=e^{i\frac{\pi}{2}}(c-o)$

$d=ic$

E صورة D وفق دوران مركزه A بزاوية $\frac{\pi}{2}$

$e-a=e^{i\frac{\pi}{2}}(d-a)$

$e-a=i(d-a)$

$e=i(ic-a)+a$

$e=-c-ia+a$

$Z_I=\frac{a+b}{2}=\frac{a+ia}{2}$

$Z_J=\frac{c+b}{2}=\frac{c+ic}{2}$

$Z_K=\frac{b+e}{2}=\frac{ic-c-ia+a}{2}$

$Z_K-Z_I=i(Z_J-a)$

$l_1=Z_K-Z_I$

$=\frac{ic-c-ia+a-a-ia}{2}$

$l_1=\frac{ic-c-2ia}{2}$

$l_2=i(Z_J-a)$

$=i\left(\frac{c+ic}{2}-a\right)=i\left(\frac{c+ic-2a}{2}\right)$

$l_2=\frac{ic-c-2ai}{2}$

تمرينات منشور ثنائي الحد:

① $S = 3^n\binom{n}{0} + 3^{n-1}\binom{n}{1} + \dots + \binom{n}{n} = (3+1)^n = 4^n$

② $S = 3^n\binom{n}{0} + 3^{n-1}\cdot 2\binom{n}{1} + 3^{n-2}\cdot 2^2\binom{n}{2} + \dots + \binom{n}{n}2^n = (3+2)^n = 6^n$

③ $11^{11} = (1+10)^{11}$ ; $T_r = \binom{11}{r}\underbrace{(1)^{11-r}}_{=1}(10)^r = \binom{11}{r}10^r$

مطلوب آحاد، عشرات ومئات: نحسب $T_0 + T_1 + T_2$

$$T_0 = \binom{11}{0}10^0 = 1$$
$$T_1 = \binom{11}{1}10^1 = 110$$
$$T_2 = \binom{11}{2}10^2 = \frac{11\times 10}{2\times 1}\times 100 = 5500$$

$$T_0 + T_1 + T_2 = 1 + 110 + 5500 = 5611$$

آحاد، عشرات ومئات العدد 611،

④ $\cos^3 x = \left(\frac{e^{ix} + e^{-ix}}{2}\right)^3 = \frac{(e^{ix} + e^{-ix})^3}{8}$

$$= \frac{e^{i3x} + 3e^{i2x}e^{-ix} + 3e^{ix}e^{-2ix} + e^{-i3x}}{8}$$

$$= \frac{e^{i3x} + 3e^{ix} + 3e^{-ix} + e^{-i3x}}{8}$$

$$= \frac{e^{i3x} + e^{-i3x} + 3(e^{ix} + e^{-ix})}{8}$$

$$= \frac{2\cos(3x) + 6\cos(x)}{8}$$

$$I = \int_0^{\pi} f(x)\,dx = \int_0^{\pi} \frac{2\cos 3x + 6\cos x}{8}\,dx$$

$$= \frac{1}{8}\int_0^{\pi} (2\cos 3x + 6\cos x)\,dx$$

$$= \frac{1}{8}\left[\frac{1}{3}2\sin 3x + 6\sin x\right]_0^{\pi}$$

$$= \frac{1}{8}\left[0 - 0\right] = 0$$

$$\lim_{x \to \frac{\pi}{2}} \frac{\cos 3x}{\cos x} : \frac{0}{0}$$

لدينا من المنشور السابق

$$\cos^3 x = \frac{2\cos 3x + 6\cos x}{8}$$

$$8\cos^3 x = 2\cos 3x + 6\cos x$$

$\cancel{8\cos^3 x}$

$$2\cos 3x = 8\cos^3 x - 6\cos x$$

$$\boxed{\cos 3x = 4\cos^3 x - 3\cos x}$$

$$\lim_{x \to \frac{\pi}{2}} \frac{\cos 3x}{\cos x} = \lim_{x \to \frac{\pi}{2}} \frac{4\cos^3 x - 3\cos x}{\cos x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\cancel{\cos x}(4\cos^2 x - 3)}{\cancel{\cos x}} = \boxed{-3}$$

④ $\sin^3 x = \left(\frac{e^{ix} - e^{-ix}}{2i}\right)^3 = \frac{(e^{ix} - e^{-ix})^3}{(2i)^3} =$

$$= \frac{e^{i3x} - 3e^{2ix}e^{-ix} + 3e^{ix}e^{-i2x} - e^{-i3x}}{-8i}$$

$$= \frac{e^{i3x} - 3e^{ix} + 3e^{-ix} - e^{-i3x}}{-8i}$$

$$= \frac{e^{i3x} - e^{-i3x} - 3(e^{ix} - e^{-ix})}{-8i}$$

$$= \frac{2i\sin(3x) - 3(2i\sin x)}{-8i}$$

$$= \frac{2i[\sin 3x - 3\sin x]}{-8i} = -\frac{1}{4}[\sin 3x - 3\sin x]$$

$$\int_0^{\frac{\pi}{2}} f(x)dx = -\frac{1}{4}\int_0^{\frac{\pi}{2}} \sin 3x - 3\sin x$$

$$= -\frac{1}{4}\left[-\frac{1}{3}\cos 3x + 3\cos x\right]_0^{\frac{\pi}{2}}$$

$$= -\frac{1}{4}\left[(0) - \left(-\frac{1}{3} + 3\right)\right] = -\frac{1}{4}\,\frac{8}{3} = \boxed{-\frac{2}{3}}$$

⑤ $\left(x + \frac{1}{\sqrt{x}}\right)^n$ $\quad a = x \quad b = \frac{1}{\sqrt{x}}$

$$T_r = \binom{n}{r} (x)^{n-r} \left(\frac{1}{\sqrt{x}}\right)^r$$

$$T_r = \binom{n}{r} x^{n-r} \, x^{-\frac{1}{2}r}$$

$$= \binom{n}{r} x^{n-\frac{3}{2}r}$$

$n - \frac{3}{2}r = 0$     متى يحوي حد ثابت

$n = \frac{3}{2}r \Rightarrow 2n = 3r$

$r = \frac{2n}{3}$

فالشرط أن يكون n مضاعف للعدد 3 .

⑥ $12^5 = (2+10)^5$ $\quad T_r = \binom{5}{r} (2)^{5-r} (10)^r$

$$T_0 = \binom{5}{0} (2)^5 (10)^0 = 32$$

$$T_1 = \binom{5}{1} (2)^4 (10) = 800$$

$$\Rightarrow T_0 + T_1 = 832$$

آحاد وعشرات العدد $\boxed{32}$

(8) $\begin{matrix} 1 & 1 \\ 2 & 2 \\ 3 & \end{matrix}$ → بطاقتين على التوالي دون اعادة

عدد النتائج التي تحمل على بطاقتين مجموعها فردي هو $\boxed{12}$

| | 1 | 1 | 2 | 2 | 3 |
|---|---|---|---|---|---|
| 1 | /// | | A | A | |
| 1 | | /// | A | A | |
| 2 | A | A | /// | | A |
| 2 | A | A | | /// | A |
| 3 | | | A | A | /// |

$\boxed{9}$ $\{A, A, A, B, B, B, B\}$

B B B

$4!$

$P_4^3$ (b4 (b3) (b2)) ((b4) (b3) (b2) (b1)) $= P_4^3 \cdot 4!$

(10) $\boxed{B}$

6 5 4 3 2 1 $= 6!$